Cost Sharing Method for a Mobile Tethering with a Coalitional Game Theory

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***Abstract*—A tethering market is a system concept in which users share the cellular bandwidth of a user by using a tethering function. In this market, the user (tethering provider) can provide his/her own bandwidth to other users (tethering users), and the tethering users defray some of the costs paid by the tethering provider to the cellular network. Lee et al. derived the users' share of the cost that maximizes the sum of each user' s satisfaction in the tethering market. However, the maximized total satisfaction cannot guarantee that all users will be satisfied with their share of the cost. In other words, some users might have some excess. The existence of users having such an excess may lead to the collapse of the tethering market. In this paper, we derive the optimal cost share, with which all tethering users are satisfied. Moreover, the conventional tethering market model assumes that all tethering users select a tethering connection instead of a cellular connection. However, if the cellular charges are not very high or if the overhead of the tethering market increases, tethering users will choose a cellular connection instead of a tethering connection. Therefore, a tethering market requires the pricing boundary to be known so that users will know when it is better to choose a tethering connection. In this paper, we show the boundary that divides the parameter areas in which the tethering/cellular connections are stably chosen.**

***Index Terms*—The mobile tethering market, Coalitional game, Core, Shapley value, Nucleolus**

I. Introduction

Tablets and smart phones have become widespread at companies, universities, and in homes, and there has been a dramatic increase in the use of the Internet. These devices have *a tethering function*, which allows phones or tablets to share their Internet connections with other devices such as laptops. The tethering function is known as "mobile hot spot", and the terminal with the tethering function such as a smart phone is known as a software access point (SoftAP), which is connected to a 3G interface and/or Wi-Fi interface [1]. Thus, users that have a SoftAP can connect to the Internet without having to pay money directly to the Internet service provider (ISP) or mobile carrier.

A market model considering these tethering environments has been proposed [2]. Fig. 1 shows this tethering market. As shown in this figure, each terminal can connect to a different cellular network, for example, a foreign Internet roaming service. If the users participate in the tethering market, they register their own information to a control server. If a user has some available bandwidth, the user (*tethering provider*) can provide his/her own bandwidth to other users (*tethering users*). On the other hand, if a tethering user needs some bandwidth, the user can obtain it by paying for it. In the tethering market, users can be either buyers or sellers depending on the situation.

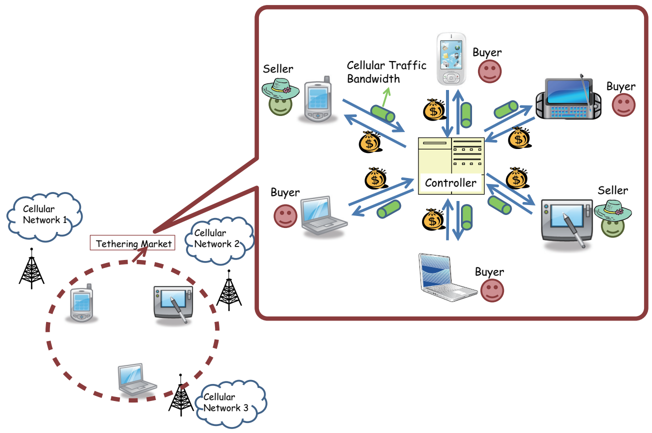

Fig. 1. An example of tethering market [3].

For example, we assume that there are four tethering users and one tethering provider in a tethering market as shown in Fig. 2. In this situation, since the tethering users defray some of the costs of the tethering provider, each tethering users' cost share needs to be configured by the tethering market system appropriately. Lee et al. derived the cost share for tethering market [2]. However, not all users are always satisfied with the derived shared cost because this cost maximize the sum of each user satisfaction [3].

Moreover, this tethering market model [2] assumes that all tethering users select a tethering connection instead of a cellular connection. However, if the cellular charges are not very high or if the overhead of the tethering market increases, tethering users will choose a cellular connection instead of a tethering connection. Therefore, a tethering market requires the pricing boundary to be known so that users will know when it is better to choose a tethering connection. We call the pricing boundary *the tethering choice boundary*.

Game theory is a very powerful tool that utilizes human behavior [4] and can be used to solve these problems. There has been a lot of research on using game theory on networks of human behavior [5]– [17]. However, almost no researches of game theory have assumed a tethering environment [7]–[17], and the results of the previous research using game theory to deal with tethering are significantly limited because this research models tethering behavior as non-cooperative games [5] [6] but does not consider cost sharing. The cost sharing is analyzed by cooperative game theory [18]– [22]. However, these cost sharing methods have not been considered in tethering environments.

From the above discussion, we can conclude that we need to analyze two major problems in the tethering environment. The main contributions of this paper are as follows.

- Modeling the cost sharing of tethering using cooperative game theory.
  - Proposing our tethering model by using cooperative game and revealing the condition for existence of imputation in Sec. II.
  - Revealing that the existence of a set of optimal shared costs considering each user's satisfaction (*the core, the Shapley value, and the nucleolus*) in Sec. III.
- Showing the boundary that divides the parameter areas in which tethering/cellular connections are stably chosen.
  - Revealing that the tethering choice boundary is a linear function of the price for cellular provider when total overhead of tethering network changes in Sec. IV.

This paper is organized as follows. Section 2 describes related studies on using game theory to study. Section 3 preliminarily explains the coalitional game. Section 4 presents our model using coalitional game theory and Section III shows the optical shared cost. Moreover, we show the boundary of the tethering choose area in Section IV. Section V shows the numerical results and concludes the paper.

II. Our proposed tethering game

A. System model

This system assumes the tethering market [2]. We assume that each user decides whether the user connects to subscribe to a cellular network or tethering market system. All users $N = \{1, \dots, n\}$ can connect to a cellular network that has bandwidth B, but each user $i \in N$ pays different costs c_i^c for the cellular network because each user has a different service provider. This payment costs is changed in following order:

$$c_1^c \geq c_2^c \geq, \dots, \geq c_{n-1}^c \geq c_n^c \geq c_0^c := 0. \quad (1)$$

When some users hope to connect to a Wi-Fi network by using the tethering market, they form a *tethering coalitional group*, which is composed of users who want to connect to the tethering market. Each user in a tethering coalitional group connects to a control server. Then each user registers the *request information* for the tethering market system, such as cellular network bandwidth B and the cost c_i^c of the plan to pay to the provider [2].

Based on the request information, the optimal tethering provider in the tethering coalitional group is selected. This optimal tethering provider subscribes the cellular provider with the smallest cost c_n^c. The other users play as the tethering users. In tethering market, all users share the tethering provider's cost c_n^c.

In order to achieve this tethering market system, we also need to derive the sharing cost c_i^t that a user i actually pays for the tethering market. This cost c_i^t is composed of the two types of costs $c_i^{t,b}$ and $c_i^{t,s}$ as follows:

$$c_i^t = c_i^{t,b} + c_i^{t,s}. \quad (2)$$

The former is the cost that the user can pay for the obtained bandwidth, and the latter is the cost that all users can pay under considering each user's value.

Denoting the number of the tethering coalitional group S as s, we assume that each user can obtain the same bandwidth $\frac{B}{s}$ as shown in Fig. 3. Here, let α be the coefficient to change from B to cost, so each tethering user or tethering provider pays the following payment:

$$c_i^{t,b} = \frac{\alpha B}{s}. \quad (3)$$

On the other hand, $c_i^{t,s}$ is decided by each user's value. Since the user will pay different payment cost c_i^c even if the cellular bandwidth of all users is B, we assume that $c_i^{t,s}$ depends on c_i^c. In this situation, we need to derive the appropriate $c_i^{t,s}$ which all users satisfied based on c_i^c. In general, it is difficult to derive the appropriate sharing cost which all users satisfied when all users have the different value. However, this cost sharing problem can apply to coalitional game theory. By using

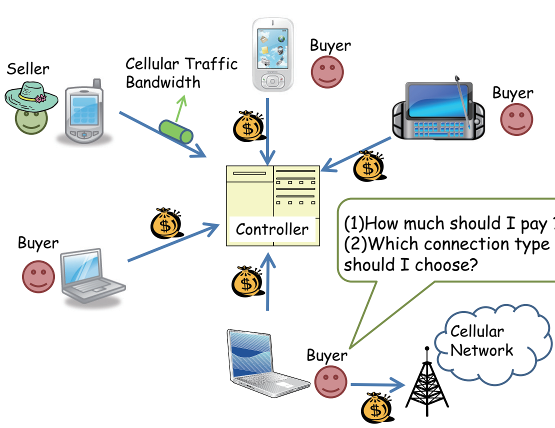

Fig. 2. Problems of the tethering market.

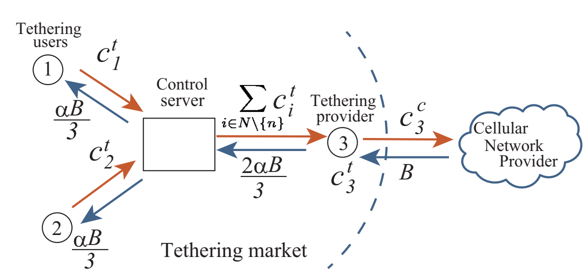

Fig. 3. System model for the tethering market (When $N = \{1, 2, 3\}$). In this situation, all users share c_3^c.

coalitional game theory, $c_i^{t,s}$ which all users are satisfied can be derived as imputation of game theory. In this paper, we derive the imputation $c_i^{t,s}$ which all users are satisfied by using coalitional game.

A cost sharing method for the coalitional game has been proposed [21] [22]. However, these coalitional games assumed that the number of sellers and buyers are fixed. In our tethering market method, the number of sellers and buyers are not fixed. Thus, we cannot apply these coalitional methods to our tethering market method. In the next subsection, we newly define our tethering market system by using a coalitional game.

B. Proposed tethering game

Let (N, v) be the proposed tethering game. Let N denote the set of players $N = \{1, ..., n\}$. A subset of N is defined as a coalition S $(\subset N)$. Let $v : 2^N \to \mathbb{R}$ be the characteristic function. We define Eq. (1) to be the cost c_i^c payable to the cellular network.

Moreover, the total overhead between tethering users increases non-linearly as the number of connecting tethering users increases [23]– [25]. Thus, we assume that the overhead $hs^p,\ h > 0,\ p > 0$ is non-linear. Note that h is the *overhead coefficient*. Thus, we can derive the characteristic function as follows:

$$v(S) = \alpha(B - hs^p) - \min_{i \in S} c_i^c, \qquad \emptyset \neq S \subseteq N. \tag{4}$$

We assume that the characteristic function of the cost [26],

$$c(S) := -v(S), \quad c : 2^N \to \mathbb{R}. \tag{5}$$

As mentioned in Subsec. II-A, we can derive $c_i^{t,s}$ as the imputation in a coalitional game. Thus, $c_i^{t,s}$ can be derived by the imputation based on Eq. (4). Moreover, we need to confirm that our tethering game satisfies the condition of a grand coalition in order to derive the imputation. In the next subsection, we derive the condition in which all users choose the tethering market (grand coalition) without an overhead ($h = 0$).

C. Condition for forming a grand coalition without overhead

To derive the imputation, a grand coalition must hold together in our tethering game. For a grand coalition to be true in this game, the characteristic function, Eq. (4), must satisfy superadditivity. However, Eq. (4) does not always satisfy superadditivity. Thus, here, we describe the condition in which the characteristic function satisfies superadditivity.

As shown in Eq. (1), c_i^c means the payment cost of user i when the user chooses the cellular network. This c_n^c is smallest in $\{c_1^c, ..., c_n^c\}$. If the user n chooses the tethering network not the cellular network, the other users $\{1, ..., n-1\}$ also choose the tethering network. The condition is as follows.

$$\alpha B \leq c_n^c. \tag{6}$$

Eq. (6) means that user n is not satisfied with the cellular cost c_n^c because the cost αB of the network bandwidth supporting the user n is greater than or equal to c_n^c. In this situation, we can consider that all users select the tethering network.

Now, let us consider a system without an overhead, in which the characteristic function is as follows:

$$v(S) = \alpha B - \min_{i \in S} c_i^c, \qquad \emptyset \neq S \subseteq N. \tag{7}$$

Here, this characteristic function of Eq. (7) means that the following inequality holds:

$$v(N) \geq v(S) \qquad \emptyset \neq S \subseteq N. \tag{8}$$

In this situation, Eq. (7) assuming Eq. (6) satisfies the superadditivity, and the grand coalition in which all users choose the tethering connection holds in Eq. (6). Thus, the inequality of Eq. (6) is the condition of the grand coalition. As explained below, we can narrow this condition $\alpha B \leq c_n$ by using proof 4.1.

Here, in this assumption of inequality, since $v(S)$ means the sum of benefits, one might feel it is strange that the sum of benefits is a negative number. Actually, this characteristic function of the tethering game includes the utility of all players by connecting them to a network. However, this total utility in $v(S)$ can be omitted by using the *strategic equivalence* [8] of cooperative games. Let β_i be the utility of player i when the player can connect to a network. The total utility of all players obtained by connecting them to a network is the sum of β_i.

This strategic equivalence means that two games (N, v) and (N, v') are equivalent if the following condition regarding the characteristic function v and v' is satisfied [27]. Let δ be a positive real number and let $\boldsymbol{\beta} = (\beta_1, \beta_2, \cdots, \beta_n)$ be a vector of real numbers. For all coalitions $S \subseteq N$, we have

$$v'(S) = \delta v(S) + \sum_{i \in S} \beta i. \tag{9}$$

Parameter δ is common in coalition S and matches both characteristic functions for the assumption of the transferable utilities [27]. Thus, the characteristic function with the total utility for network connection is as follows:

$$v(S) = \sum_{i \in S} \beta_i + \alpha B - \min_{i \in S} c_i^c, \qquad \emptyset \neq S \subseteq N.$$

Note that we use the characteristic function omitting the total utility of the network connection.

We show that the superadditivity holds with the following condition:

Theorem II.1. *A tethering game (N, v) in which the characteristic function $v(S)$ is equal to $\alpha B - \min_{i \in S} c_i^c$ is superadditive when the following equation holds:*

$$\alpha B \leq c_{n-1}^c. \tag{10}$$

Theorem II.1 shows that a condition of superadditive is sufficient to check only the difference between the cost for network bandwidth αB and the cost of user $n-1$ for cellular network c_{n-1}^c. We confirmed that our tethering model satisfies the grand coalition in Theorem II.1. The next subsection describes a closed form solution of some imputations — core, Shapley value, and nucleolus — of the tethering game.

III. Cost sharing considering each user satisfaction

A. The core of our tethering game

The airport game [28] [29] is similar to the tethering game. The characteristic function $v_{\mathrm{air}}(S)$ of the airport game is the maximum cost $v_{\mathrm{air}}(S) = -\max_{i\in S} c_i$. In general, a convex game which is characterized has a unique stable set of imputations that coincides with its core [30]. The airport game is a convex game [28], but the tethering game is not for the following reasons.

Theorem III.1. *The tethering game (N, v) when the characteristic function is $v(S) = \alpha B - \min_{i\in S} c_i$ is not a convex game.*

However, we can prove that the core is not empty by using group rationality and coalitional rationality.

Theorem III.2. *The core of the tethering game (N, v) when the characteristic function $v(S) = \alpha B - \min_{i\in S} c_i$ is not empty.*

The existence of the core means the existence of the imputation with which all users are satisfied. However, the core is not an optimal solution because there are some imputations satisfying the core. In the next subsection, we show the Shapley value of the tethering model. The Shapley value can derive an imputation in accordance with the marginal contribution of each user.

B. The Shapley value of our tethering game

The tethering game (N, v) without overhead is superadditive. Thus, the Shapley value satisfies the imputation I. The Shapley value means that each player should be paid in accordance with how valuable his/her cooperation is to the other players [31]. Thus, the Shapley value of our tethering model means the sharing costs according to onefs own payable costs.

The Shapley value is characterized by four axioms by which we can derive the Shapley value as follows.

Theorem III.3. *The Shapley value $\boldsymbol{\phi}(v) = (\phi(v)_1, \phi(v)_2, \cdots, \phi(v)_n) \in \mathbb{R}^n$ of the tethering game (N, v) when the characteristic function $v(S)$ is $\alpha B - \min_{i\in S} c_i$, $c_0 = 0$ is defined as,*

$$\phi(v)_j = -\sum_{i=1}^{j} \frac{-\frac{\alpha B}{n} + c_i - c_{i-1}}{n - i + 1}, \qquad j \in N. \tag{11}$$

Thus, we showed the solution of the Shapley value. In the next subsection, we show the nucleolus of the tethering model. The nucleolus is an efficient imputation that successively minimizes the largest excess.

C. The nucleolus ν of our tethering game

From the definition of the nucleolus, the excess of the nucleolus is defined as

$$e^*(S, v) \quad := \quad \min_{x\in I} \max_{S\in N} (\alpha B - \min_{i\in S} c_i - \sum_{i\in S} x_i), \ \forall S \in N.$$

The nucleolus means a set of imputations that minimizes the maximum excesses of the coalition. To derive the nucleolus, the excesses of two coalitions are compared with the lexicographic ordering [27]. When vector x is larger than vector y in lexicographic ordering, we have $x_i = y_i, i = 1, ..., k-1$ and $x_k > y_k$ in some index $k = 1, ..., K$. By using this assumption, our tethering game can formulate the following equation.

Definition III.1. *The nucleolus $-\boldsymbol{\nu} := \mathbf{y}^* = (y_1^*, y_2^*, ..., y_n^*)$ of tethering game (N, v) can be defined as the following equations:*

$$y_1^* = ... = y_{n-1}^* = \frac{c_{n-1} - \alpha B}{n} = r^*,$$
$$y_n^* = c_n - (n-1)r^*. \tag{12}$$

In general, the nucleolus is included in the core when the core is not empty. Thus, we can derive the solution in which all users are satisfied and the largest excess is minimized.

We have derived the existence of the core, Shapley value, and nucleolus for the tethering model without overhead. However, there is generally a trade-off between the obtained bandwidth (throughput) and the payment cost. From the viewpoint of users, if the payment cost is low, the users will choose the tethering connection even if their own bandwidth is small. Thus, we should consider the relationship between overhead and the payment cost.

In the next section, we explain the characteristic of our tethering model with an overhead. By analyzing the model, we assess the trade-off between bandwidth (throughput) and costs.

IV. Existence of superadditivity with an overhead

This section shows the relationship between the overhead with tethering and the payment cost for cellular provider. In what follows, we refer to the number of elements of two coalitions as s, t such that $s + t = n$, $\mathbb{Z} \ni s \geq 1$, $\mathbb{Z} \ni t \geq 1$, $n \in \mathbb{Z}$. In the first step, we proof the following Lemma.

Lemma IV.1. *When $p > 1$, the following s and t maximize $n^p - s^p - t^p$:*

$$s = \lfloor \frac{n}{2} \rfloor, t = \lceil \frac{n}{2} \rceil.$$

By using this Lemma, we can derive the relationship between the overhead and the payment cost.

Theorem IV.1. *When the characteristic function $v(S) = \alpha B - \alpha h s^p - \min_{i\in S} c_i$ ($\alpha \geq 1,\ p > 1$) in the tethering game (N, v) is superadditive and the cost for each user is assumed to be*

$$c_i := c_1 = c_2 = c_3, ... = c_{n-1} = \xi c_n > c_0 = 0, \quad \xi > 1,$$

the following inequality holds:

$$\frac{c_{n-1}}{\alpha B} \geq \frac{n^p - (\lfloor \frac{n}{2} \rfloor^p + \lceil \frac{n}{2} \rceil^p)}{n^p} h_{tg} + 1.$$

$$h_{tg} := \frac{hn^p}{B}, \quad where \quad \frac{c_{n-1}}{c_n} > 1.$$

Moreover, we can derive the tethering choice boundary when $0 < p < 1$.

Theorem IV.2. *When the characteristic function $v(S) = \alpha B - \alpha h s^p - \min_{i \in S} c_i$ ($\alpha \geq 1,\ 0 < p < 1$) in the tethering game (N, v) is superadditive and the cost for each user is assumed to be*

$$c_i := c_1 =, \ldots = c_{n-1} = \xi c_n > c_0 = 0,\ \xi > 1, \quad (13)$$

the following inequality holds:

$$\frac{c_{n-1}}{\alpha B} \geq \frac{2^p - 2}{n^p} h_{tg} + 1, h_{tg} = \frac{hn^p}{B}, \quad where \quad \frac{c_{n-1}}{c_n} > 1.$$

Therefore, as discussed in Subsec. II-C, to check a condition of superadditive with overheads, the cost with user $n-1$ / the cost that the user can accept to pay for network bandwidth ratio $\frac{c_{n-1}}{\alpha B}$ is a criterion value in Theorem IV.1 and IV.2.

V. Numerical results

A. The core, The Shapley value and the nucleolus

We numerically solved our tethering model. We assumed that there were three players $\mathcal{N} = \{1, 2, 3\}$ and $\mathbf{c} = (c_1, c_2, c_3)$. Let $\mathbf{c}$ be the cost for each player per day. First, we set $\alpha = 0$ to confirm a basic characteristic. Note that $c_i^{all} = c_i^{satisfy}$. Let $e^{\phi}(S, v)$ be the maximum excess of the Shapley value and $e^*(S, v)$ be the maximum excess of the nucleolus.

As mentioned in Subsec. II-A, $c_i^{satisfy}$ can be derived by imputation of $v(S)$. Note that the shared cost in $\boldsymbol{\phi}$ and $\boldsymbol{\nu}$ is minus according to Eq. (5). The Shapley value $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ was allocated in accordance with each user's contribution as shown in Subsec. III-B. In this case, the imputation was determined by each user's cost to be paid. For example, when $\mathbf{c} = (30, 15, 12)$[dollar], player 3 has the lowest cost. In this situation, the Shapley value $\boldsymbol{\phi} = (-10, -25, 5)$ and the nucleolus $\boldsymbol{\nu} = (-5, -5, -2)$. Thus, if all players cooperate in tethering and share the cost in accordance with the Shapley value, player 3 receives 5[dollar] from Players 1 and 2. Players 1 and 2 pay in accordance with ϕ_1 and ϕ_2. Since $e^{\phi}(S, v) = -5(< 0)$, the nucleolus is in the core.

Moreover, we set $\alpha = 2$ and $B = 6$, and we calculate the cost $c_i^{all} = c_i^{band} + c_i^{satisfy}$ when $\mathbf{c} = (30, 15, 12)$. In this situation, if the tethering users and tethering provider obtain the same bandwidth, $c_i^{band} = 4$ because $c_i^{satisfy}$ can be derived by the Shapley value and the nucleolus. The Shapley value and the nucleolus are $\boldsymbol{\phi} = (-8.67, -0.67, 9.33)$ and $\boldsymbol{\nu} = (-2, -2, 4)$, respectively. Thus, $\mathbf{c}^{all} = (-12.67, -4.67, 5.33)$ when $c_i^{satisfy}$ is calculated by $\boldsymbol{\phi}$, and $\mathbf{c}^{all} = (-6, -6, 0)$ when $c_i^{satisfy}$ is calculated by $\boldsymbol{\nu}$. Note that the cost for the tethering provider is $c_n = 12$ in both situations.

Fig. 4 shows the limitation of stable solutions in every coalition. Each vertex of the triangle means the imputation with one coalition, and the height of the triangle means the coalitional value of a grand coalition $v(N)$. Note that we use *zero-normalized* game [32] to form strategic equivalence for this figure in Eq. (9). The zero-normalized game (N, v_0) is defined by the following equation.

$$v_0(S) = v(S) - \sum_{i \in S} v(\{i\}), \qquad \forall S \subseteq N. \quad (14)$$

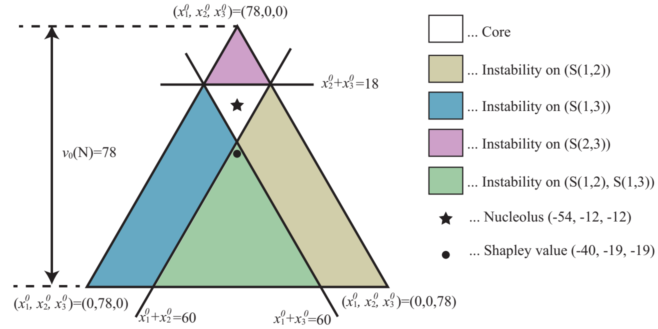

Fig. 4. Limitation of stable solutions in our tethering model.

When the game is a zero-normalized game, let $\mathbf{x}^0 = \{x_1^0, x_2^0, \ldots, x_n^0\}$ be the imputations of players. We set $n = 3$, $\mathbf{c} = (60, 18, 12)$, $\alpha = 0$. By using Eq. (14), $v_0(N)$ is set to 78. Moreover, $v_0(1, 2) = v_0(1, 3) = 60$, and $v_0(2, 3) = 18$. The area of the core is judged. As shown in Fig. 4, the Shapley value does not include the core. Thus, if the players share the tethering costs in accordance with the Shapley value (in other words, if the players share the costs in accordance with the contribution of each player), some players cannot be satisfied by the value of cost sharing.

VI. Conclusion

In this paper, we modeled cost sharing of a mobile tethering environment using coalitional games of cooperative game theory. We derived the existence of a set of optimal shared costs, which means that the tethering provider and all tethering users are satisfied. Our quantitative study showed the boundary at which users should choose to be tethering users or not. Numerical results showed our derived shared costs and the relationship between the payment cost for cellular provider and the overhead of the tethering network.

This paper assumed that the users act reasonably, which does not always happen in reality. In future work, we will analyze user behavior considering bounded rationality in the mobile tethering environment.

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