Reward Distribution Using an Anti-duality Game in a Point Cloud Data Trading Model Based on Collected Area

Jin Watanabe
College of Engineering
Shibaura Institute of Technology
Tokyo, Japan
af20011@shibaura-it.ac.jp

Sumiko Miyata
College of Engineering
Shibaura Institute of Technology
Tokyo, Japan
sumiko@shibaura-it.ac.jp

Kenji Kanai
Waseda Research Institute for Science and Engineering
Waseda University
Tokyo, Japan
k.kanai@aoni.waseda.jp

***Abstract*—Digital Twin is expected to be a key technology for realizing the next-generation smart city. It can be applied to social infrastructure, manufacturing, and medical fields. This paper focuses on "Digital Twin City", which collects all kinds of data in a city and uses it for public services such as transportation, disaster prevention and urban planning, thereby improving the quality of life for residents. Three-dimensional urban landscapes formed by point cloud data are essential for the development of public services and other services in Digital Twin City. However, installing sensors and collecting fresh data from every corner of the city involves high initial and maintenance costs. Therefore, involving residents in data collection is considered a promising option. To create incentives for residents to participate as data providers, we propose a data trading cooperation game based on coalitional game. As a characteristic function of the game, an anti-duality game of airport game that solves the already known cost allocation problem is used, which corresponds to the reward distribution problem in this paper. In the numerical example, the distribution of reward is determined around Nucleolus, which is the allocation that minimizes the maximum excess of data providers.**

***Index Terms*—Digital Twin, Point cloud, Anti-duality game, Nucleolus**

I. Introduction

Digital Twin is expected to be a key technology [1] for realizing the next-generation smart city. Digital Twin enables the creation of highly accurate digital models in a virtual space in real time by sensing data from an actual environment. The digital model allows us to achieve a very realistic simulation (mirroring of the real world), which can be used to solve various social problems. Digital Twin can be applied to social infrastructure, manufacturing, and medical fields. In particular, we focus on Digital Twin City as described in [1] — collecting all kinds of data in a city and using them for public services such as transportation, disaster prevention, and city planning to improve the quality of residents' lives.

Digital Twin City has tried to implement and demonstrate worldwide, including Virtual Singapore [2]. In Japan, Virtual SHIZUOKA [3], the PLATEAU Project [4] of the Ministry of Land, Infrastructure, Transport and Tourism, and the Tokyo Metropolitan Government's Tokyo Digital Twin Project [5] are currently testing. To create a Digital Twin City, all the city's latest spatio-temporal data are collected in real-time, including three-dimensional city scenery and city activities such as weather information, public transportation, and human activities.

Three-dimensional city scenery formed by point cloud data is essential for launching public and other services in Digital Twin City. However, collecting fresh data on every city corner by setting up sensors would require high initial and maintenance costs.

The authors of this paper proposed the concept of a Co-creation of a Digital Twin City by letting residents get involved in data collection and creating a Digital Twin City [6] [7] (in Fig.1). However, there is a lack of consideration of how to encourage residents to participate as data providers. Data consumers — companies that use the collected data — should cooperate to create incentives to give back to data providers.

We believe the cooperative game theory is a practical solution to the above problem. In cooperative game theory, several solutions have been proposed for distributing profits among players, including core, Shapley value, and Nucleolus. Among them, the solution called Nucleolus has attracted attention. Nucleolus is the most stable solution because it lexicographically minimizes the dissatisfaction score among all coalitions. It has been proposed as a method of cost allocation and reward distribution.

The main problem in cooperative game theory is that the cost allocation assumes an indirect distribution of reward. In addition, it did not consider excess, which is a measure of dissatisfaction of the data providers.

Therefore, this paper proposes a data trading cooperation game using coalitional games to create incentives for residents to participate as data providers. The proposal can be a pricing design for point cloud data trading in the smart city [8]. The coalitional games, the basic model of cooperative game theory, is widely known as a theory that discusses how to form a coalition and distribute profits to each player in the presence of multiple autonomous players [9].

Using an anti-duality game in cooperative game theory, we

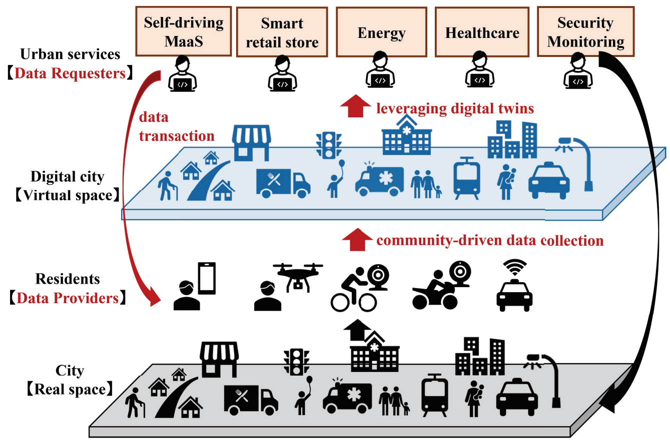

Fig. 1. Overview of the Co-creation of a Digital Twin City.

formulate the problem as a reward distribution. The amount of reward distribution to each data provider is determined with a focus on the distribution that minimizes the maximum excess.

II. Related Work and Game theory

A. Related Work

Mobile Crowd Sensing (MCS) is a new sensing network concept that enables low-cost, wide-area sensing by mobile users using their smartphones as sensors [10]. There are two different MCS models, depending on who decides the task scheduling (the server or each user). In this paper, we focus on User-centric Participatory Sensing (UPS), in which each participating user decides individually on task scheduling and determines the task to be performed.

In UPS, this is often done decentralized with local information. Jiang et al. proposed a P2P-based MCS system [11] [12]. They also focused on task similarity and proposed a method to improve the overall system efficiency by reusing data among different tasks [13]. However, these studies are based on a one-to-one correspondence between buyers and sellers of data, and they do not take into account the characteristics of the sensing data concerning its price design, such as the fact that superimposition improves the quality of the data, as in the case of point cloud data.

In the cost above allocation [14], focusing on data consumers who buy data, the share of expenses covered by each consumer is calculated for cases in which multiple data consumers who meet certain conditions cooperate to purchase data. However, the paper does not take data price into account. Therefore, this paper designs pricing considering where point cloud data is collected and where multiple providers cooperate to sense such data in requested areas. Then the amount of reward to each data provider is calculated based on cooperative game theory [9] [15].

B. Coalitional Game

A coalitional game can determine the amount of reward for each data provider. In a coalitional game, $\mathcal{N} = \{1, 2, \ldots, n\}$ is the set of n players, and a subset $\mathcal{S}$ of $\mathcal{N}$ is called a coalition of players ($s = |\mathcal{S}|$). For each coalition, the function v corresponding to the worth of the coalition is called the characteristic function of the game, and $v(\mathcal{S})$ is here called the coalition value. A game $G(\mathcal{N}, v)$ formulated in terms of the set $\mathcal{N}$ and the characteristic function v is called a coalitional game.

In the game $G(\mathcal{N}, v)$, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is called the payoff vector of the game. When this payoff vector is an imputation, it must satisfy the two properties of collective rationality and individual rationality [9]. The imputation is a payoff vector that satisfies that each player receives more payoff than he or she could have earned alone and that the sum of the payoffs received by the players is the payoff earned by all.

Collective Rationality

When a coalition is formed by all n players, the coalition value $v(\mathcal{N})$ needs to be distributed among all the players. Therefore, it must satisfy $\sum_{i=1}^{N} x_i = v(N)$. This requirement on x_i is called collective rationality.

Individual Rationality

The payoff for a player i who does not join a coalition is $v(i)$, and if he does join a coalition, player i would demand at least $v(i)$ as his share of the payoff. Therefore, if $x_i \geq v(i)$ is satisfied, x_i is said to possess individual rationality.

The core of the game is the set of payoffs for which there is no unsatisfactory coalition, and is known to coincide with payoffs that satisfy the following condition of coalitional rationality [9].

$$\sum_{i \in \mathcal{S}} x_i \geq v(\mathcal{S}) \ , \forall \mathcal{S} \in \mathcal{N}. \tag{1}$$

For any coalition, the sum of its payoffs is greater than the values of the coalition. The core is a set of imputations, however, typical examples of solutions to the coalitional game are uniquely determined values such as the Shapley value and the Nucleolus.

1) Shapley Value: First, we discuss Shapley values. In a coalition $\mathcal{S}$, a player i is not included in $\mathcal{S}$. The players of the coalition $\mathcal{S}$ can get $v(\mathcal{S})$, however when player i joins this coalition, the value of coalition is $v(\mathcal{S} \cup \{i\})$. When a player i joins a coalition, the difference between the characteristic function values with $\mathcal{S}$ and the value with $\mathcal{S} - \{i\}$ is called the marginal contribution of player i to the coalition $\mathcal{S}$, and its average is called the Shapley value [9]. In a coalitional game $G(\mathcal{N}, v)$ with transferable utility, the Shapley value of player i is defined by the following equation.

$$x_i^{\mathrm{shp}} = \sum_{i \in \mathcal{S} \subset \mathcal{N}} \frac{(s-1)!(n-s)!}{n!} \{v(\mathcal{S}) - v(\mathcal{S} - \{i\})\}. \tag{2}$$

The formula of $(s-1)!(n-s)!/n!$ represents the probability that player i join the coalition $\mathcal{S} - \{i\}$ in the process of forming the entire coalition $\mathcal{N}$ with n players in random order [16].

2) Nucleolus: For a coalition $\mathcal{S}$, the difference between the sum of a payoff $\sum_{i \in \mathcal{S}} x_i$ and the coalition value $v(\mathcal{S})$ has the excess that the coalition $\mathcal{S}$ has with the payoff vector $\mathbf{x}$ as the following formula.

$$e(\mathcal{S}, \mathbf{x}) := v(\mathcal{S}) - \sum_{i \in \mathcal{S}} x_i. \tag{3}$$

When the excess is large, the sum of each player's payoffs is greater than the coalition value. In this situation, these players do not want to cooperate with each other in $\mathcal{S}$. Let $\boldsymbol{\theta}(\mathbf{x})$ be a vector of excess vectors for the excesses of all coalitions in order of their size. Next, we consider two vectors $a = (a_1, \ldots, a_K)$ and $b = (b_1, \ldots, \ldots, b_K)$. Now, a vector a is greater than a vector b in the sense of lexicographic order if for some $k = 1, 2, \ldots K$ exists and the following equation holds.

$$a_i = b_i, \quad i = 1, \ldots, k-1 \quad \text{and} \quad a_k > b_k. \tag{4}$$

In this case, we write $a >_L b$. Therefore, for a payoff vector $\mathbf{x}$ and an payoff vector $\mathbf{y}$, $\mathbf{y}$ is more acceptable than $\mathbf{x}$ if $\boldsymbol{\theta}(\mathbf{x}) >_L \boldsymbol{\theta}(\mathbf{y})$. The Nucleolus of a game is an imputation for which there is no more acceptable payoff [9]. Nucleolus belongs to the core if the core is non-empty. This is due to $\max_{\mathcal{S}} e(\mathcal{S}, \mathbf{y}) \leq 0$ for the imputation $\mathbf{y}$ belonging to the core [15].

Cooperative game theory is also used for cost allocation and reward distribution problems; Littlechild proposed the airport game [17]. The cost of constructing a runway is determined by the aircraft with the longest runway among the aircraft using it. The cost of a runway is based on the length of a runway. Thus, the maximum cost of a runway is shared by coalitional players.

III. Proposal Method

A. Use Case

Incentive mechanisms are categorized into two categories: reward-sharing mechanism and auction-based mechanism [18]. The reward-sharing mechanism is that a data consumer pays rewards to the contributors (e.g., data providers), and the reward is allocated among contributors according to the degree of their contributions. The mechanism is applied to the use case of cooperative tasks. On the other hand, the auction-based mechanism is that the data consumer offers his/her task at auction, and the data providers try to make a winning bit. The mechanism is applied to the competitive use case.

This paper adopts the reward-sharing mechanism for the pricing design (e.g., incentive mechanism) of the Co-creation of a Digital Twin City. A concept of the proposed pricing design is illustrated in Fig. 2. Our mechanism consists of multiple data consumers and data providers. The data consumers are requesting point cloud data of a city, and in this case, all of them are asking for data in the same area. Furthermore, data providers, who are residents of the city, expect to receive rewards by sensing point cloud data. In addition, we assume that data providers preform sensing in the requested area, whereas the extent of the area covered by each provider is different. When data consumers purchase the sensed data, an additional reward is provided for the improved quality of the point cloud data made possible by the overlapping data of the providers in the same area. Since a coalition of data providers receive reward for sensing, this paper determines the allocated amount gained by each provider under this use case.

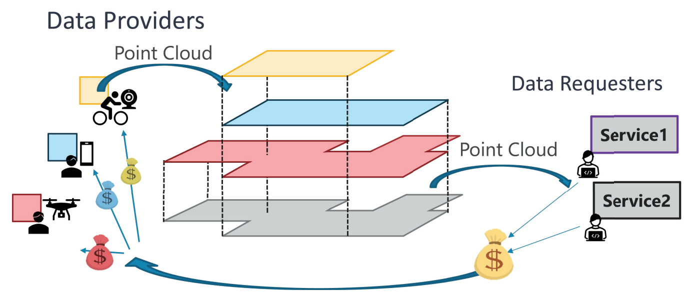

Fig. 2. Overview of price design.

B. Data Transaction Model

The key notations in this paper are listed in Table I. The set of data providers collecting point cloud data is defined as $\mathcal{N} := \{1, 2, \ldots, n\}$. Let α_i be the ratio of the area to the requested area collected by the i-th data provider, and assume that $\alpha_1 > \alpha_2 > \ldots > \alpha_n > \alpha_{n+1} := 0$. The value of the $n+1$-th user is defined as 0. Let $\mathcal{S}$ be the set of coalition, meaning cooperative groups in the game, and define the characteristic function of the game by the following equation. Note that this equations are characteristics function for anti-duality game of airport game [19].

$$v(\mathcal{S}) = \begin{cases} \alpha_1 - \max_{j \notin \mathcal{S}} \alpha_j & \text{if } 1 \in \mathcal{S} \subseteq \mathcal{N}, \\ 0 & \text{if } 1 \notin \mathcal{S}. \end{cases} \tag{5}$$

where $v(\emptyset) = 0$. The coalition value for $\mathcal{S}$ involving player $\{1\}$ is defined as the total payoff minus the maximum area ratio of the players outside the coalition, i.e. the incremental area ratio compared to outside the coalition. By considering the outside the coalition, the influence of players who do not participate in the coalition is taken into account. Finally, coalitions without player $\{1\}$ have a payoff of zero because there is no increment.

When a game has an anti-duality game, the core, Nucleolus, Shapley value of anti-duality game are obtained by reversing

TABLE I
Key Notations.

Symbols	Physical Meaning
$\mathcal{N} = \{1, 2, \ldots, n\}$	Set of data providers
$\mathcal{S} \subseteq \mathcal{N}$	Set of coalition of data providers
$v(\mathcal{S})$	Characteristic function
$\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$	Vector of area ratio covered by each player
$\mathbf{x} = (x_1, x_2, \ldots, x_n)$	Vector of reward sharing ratio
$\mathcal{R} = \{1, 2, \ldots, r\}$	Set of data consumers
$\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_r)$	Vector of consumers' value with respect to data
T	Total reward
$p \times \sum_{i \in \mathcal{N}} \alpha_i$	Quality-based pricing
η	Revenue sharing factor
s	Transmission cost

the positive and negative values of the respective values of the original game. Our system model is similar to the airport game. However, this game has contrasts in terms of cost allocation and reward distribution. In general, relationship between cost allocation problem and reward distribution problem is anti duality game. Therefore, we use an anti duality game for our reward distribution problem.

1) Shapley Value: The Shapley value $\mathbf{x}^{\mathrm{Shp}}$ is given by the following equation.

$$x_i^{\mathrm{Shp}} = \sum_{i=j}^{n} \frac{\alpha_i - \alpha_{i+1}}{i}, \quad j \in \mathcal{N}. \tag{6}$$

2) Nucleolus: The Nucleolus can minimize the maximum excess that the data provider feels about the imputation, which is obtained as $\mathbf{x}^{\mathrm{Nu}} = (x_1^{\mathrm{Nu}}, ..., x_n^{\mathrm{Nu}})$ using the following equation [15].

$$\begin{aligned}
&x_i^{\mathrm{Nu}} = r_{n+1-k} \text{ for} k = 1, \ldots, k' \text{ and } i_{n+k-1} \le i < i_{n+1-(k-1)} \\
&\quad \text{with} \quad i_{n+1} := n+1, \quad i_{n+1-k'} := 1, \quad r_{n+1} := 0 \\
&\quad \text{where:} \\
&r_{n+1-k} = \min\left(\min_{2,\ldots,i_{n+1-(k-1)}-1} \left[\frac{\alpha_i - \alpha_{i_{n+1-(k-1)}} + r_{n+1-(k-1)}}{i_{n+1-(k-1)} - i + 1} \right], \right. \\
&\qquad\qquad \left. \frac{\alpha_1 - \alpha_{i_{n+1-(k-1)}} + r_{n+1-(k-1)}}{i_{n+1-(k-1)} - 1} \right) \\
&i_{n+1-k} = \min\{\arg\ \min(\cdot)\} \quad \text{for } k = 1, \ldots, k'
\end{aligned} \tag{7}$$

where $i = 1, 2, ...n, n+1$. Let $\min[\cdot]$ be the minimum value for i that satisfies $2 \le i \le i_{n+1-(k-1)} - 1$, and i_{n+1-k} be the minimum $i \in \mathcal{N}$ that achieves $\min(\cdot)$ for each of $k = 1, ..., k'$. For a detailed derivation of Shapley values and Nucleolus, see the original airport game [17] [19].

3) Reword Paid by Data Consumers: Let $\mathcal{R} := \{1, 2, ..., r\}$ be a set of data consumers who purchase point cloud data. The total amount of reward T paid by the data consumer to the data provider is given by the following equation, referring to the conventional price design [12],

$$T = \sum_{j \in \mathcal{R}} [p \sum_{i \in \mathcal{N}} (\alpha_i) + \eta(\beta_j - s)]. \tag{8}$$

In Eq. (8), p refers to a fixed price for data in the total requested area, η is a revenue sharing coefficient used when data consumers return a part of the profit to data providers, β_j is the value of the data that differs by data provider j, and s is the data transmission cost. The first term corresponds to a revenue sharing scheme and the second term to a quality-based pricing scheme, both of which are widely used in the literature [11] [12] [20]. This pricing method includes as a special case both pure wholesale pricing ($p = 0$) and pure user-based pricing ($\eta = 0$), and can correspond to a variety of scenarios. In general, the quality of point cloud data improves if the areas they are taken overlap in Fig. 2. Therefore, in this paper, we introduce a novel idea of this overlap for point cloud data. This idea is given by $\sum_{i \in \mathcal{N}} (\alpha_i)$ which means that the more rewards can be paid to the data provider when the provider get larger area then other providers.

The amount of rewards $\mathbf{y}$ obtained by each data provider is determined as follows,

$$\mathbf{y} = T \times \mathbf{x}. \tag{9}$$

The total reward amount T is multiplied by the payoff vectors $\mathbf{x}$, which are the reward distribution ratios determined by game theory.

IV. Numerical Example

In the numerical example, we assume that three data providers participate in data collection (n=3). In this case, we present numerical analysis examples for two percentages of the requested area that each data provider sensed, $\boldsymbol{\alpha} = (1, 0.5, 0.2)$ and $\boldsymbol{\alpha} = (1, 0.2, 0.1)$. We analyze the excess of each payoff vector according to differences in the size of the area of point cloud data collected by each data provider. Note that smaller areas are included in larger areas. Further weighting of importance within a requirement area is needed, but investigating such complex alliances is a subject for future work.

In this section, $\mathbf{x}^{\mathrm{ratio}}$ for the payoff vectors determined by the ratio of $\boldsymbol{\alpha}$, $\mathbf{x}^{\mathrm{shp}}$ for the payoff vectors determined by the Shapley value, and the payoff vectors determined by Nucleolus are $\mathbf{x}^{\mathrm{nu}}$, and $\mathbf{x}^{\mathrm{ave}}$ for the payoff vectors determined by equal sharing.

A. Example 1-1 : $\boldsymbol{\alpha} = (1, 0.5, 0.2)$

The payoff vectors determined by the ratio of $\boldsymbol{\alpha}$ and the payoff vectors determined by equal sharing are respectively as follows.

$$\mathbf{x}^{\mathrm{ratio}} = (0.588, 0.294, 0.118), \quad \mathbf{x}^{\mathrm{ave}} = (0.333, 0.333, 0.333).$$

The payoff vectors for Shapley value and Nucleolus are as follows from the (6) and (7), respectively.

$$\mathbf{x}^{\mathrm{shp}} = (0.717, 0.217, 0.067), \quad \mathbf{x}^{\mathrm{nu}} = (0.7, 0.2, 0.1).$$

These three payoff results $\mathbf{x}^{\mathrm{ratio}}$, $\mathbf{x}^{\mathrm{shp}}$, and $\mathbf{x}^{\mathrm{nu}}$ that satisfy individual rationality and collective rationality are imputation. On the other hand, $\mathbf{x}^{\mathrm{ave}}$ does not satisfy imputation condition. The excess of each coalition over its payoff is defined as Eq. (3). A list of the payoff for each coalition $\mathcal{S}$ and the resulting excess is shown in Table II.

TABLE II
Excesses for each payoff at $\boldsymbol{\alpha} = (1, 0.5, 0.2)$.

Coalition $\mathcal{S}$	$v(\mathcal{S})$	$e(\mathcal{S}, \mathbf{x}^{\mathrm{nu}})$	$e(\mathcal{S}, \mathbf{x}^{\mathrm{shp}})$	$e(\mathcal{S}, \mathbf{x}^{\mathrm{ave}})$	$e(\mathcal{S}, \mathbf{x}^{\mathrm{ratio}})$
{1}	0.5	−0.2	−0.217	0.167	−0.088
{2}	0	−0.2	−0.217	−0.333	−0.294
{3}	0	−0.1	−0.067	−0.333	−0.118
{1, 2}	0.8	−0.1	−0.133	0.133	−0.082
{1, 3}	0.5	−0.3	−0.283	−0.167	−0.206
{2, 3}	0	−0.3	−0.283	−0.667	−0.412
{1, 2, 3}	1	0.0	0.0	0.0	0.0

Based on the excess of each coalition shown in Table II, we show the excess vectors $\boldsymbol{\theta}(\mathbf{x})$. The excess vector is the excess $e(\mathcal{S}, \mathbf{x})$ of each coalition in increasing order.

$$\boldsymbol{\theta}(\mathbf{x}^{\text{ratio}}) = (0.0, -0.082, -0.088, -0.118, -0.206, -0.294, -0.412)$$
$$\boldsymbol{\theta}(\mathbf{x}^{\text{shp}}) = (0.0, -0.067, -0.133, -0.217, -0.217, -0.283, -0.283)$$
$$\boldsymbol{\theta}(\mathbf{x}^{\text{nu}}) = (0.0, -0.1, -0.1, -0.2, -0.2, -0.3, -0.3)$$
$$\boldsymbol{\theta}(\mathbf{x}^{\text{ave}}) = (0.167, 0.133, 0.0, -0.167, -0.333, -0.333, -0.667)$$

Comparing the four excess vectors in the lexicographic order defined in (4), we obtain $\boldsymbol{\theta}(\mathbf{x}^{\text{ave}}) >_L \boldsymbol{\theta}(\mathbf{x}^{\text{shp}}) >_L \boldsymbol{\theta}(\mathbf{x}^{\text{ratio}}) >_L \boldsymbol{\theta}(\mathbf{x}^{\text{nu}})$.

Since all the elements of the excess vectors $\boldsymbol{\theta}(\mathbf{x}^{\text{ratio}})$, $\boldsymbol{\theta}(\mathbf{x}^{\text{shp}})$, and $\boldsymbol{\theta}(\mathbf{x}^{\text{nu}})$ are less than or equal to 0, that is, Eq. (1) is satisfied, the three imputations belong to the core, we see that Nucleolus achieves the distribution with the least excess among the three imputations. Note that the excess in $\mathbf{x}^{\text{ave}}$ is positive in the coalition $\{1\}, \{1, 2\}$. This means that the coalition $\{1\}$ is dissatisfied with the payoff of $\{2, 3\}$, and the coalition $\{1, 2\}$ is dissatisfied with the payoff of $\{3\}$. Since the excess is positive, we can also say that the coalition rationality is not satisfied ($x_1 \geq v(\{1\})$). In other words, this payoff vector does not belong to the core. Therefore, the allocation of reward by $\mathbf{x}^{\text{ave}}$ is not appropriate.

B. Example 1-2 : $\boldsymbol{\alpha} = (1, 0.2, 0.1)$

The following is an example of numerical analysis for the case where the range of the collected area of each player is biased. The payoff vectors determined by the ratio of $\boldsymbol{\alpha}$ and the payoff vectors determined by by equal sharing are respectively as follows.

$$\mathbf{x}^{\text{ratio}} = (0.769, 0.154, 0.077), \quad \mathbf{x}^{\text{ave}} = (0.333, 0.333, 0.333).$$

The payoff vectors for Shapley value and Nucleolus are as follows from the Eqs. (6) and (7), respectively.

$$\mathbf{x}^{\text{shp}} = (0.883, 0.083, 0.033), \quad \mathbf{x}^{\text{nu}} = (0.875, 0.075, 0.050).$$

The two payoffs $\mathbf{x}^{\text{shp}}$ and $\mathbf{x}^{\text{nu}}$ that satisfy individual rationality and collective rationality are imputation. In this example, the payoff vectors $\mathbf{x}^{\text{ratio}}$ by collected area, which is the ratio of α, are not imputation. Moreover, $\mathbf{x}^{\text{ave}}$ does not satisfy the imputation condition. A list of the payoffs for each coalition $\mathcal{S}$ and the resulting excess is given in Table III.

Based on the excess of each coalition shown in Table III, we show the excess vectors $\boldsymbol{\theta}(\mathbf{x})$. The excess vector is the excess $e(\mathcal{S}, \mathbf{x})$ of each coalition in increasing order.

TABLE III
EXCESSES FOR EACH PAYOFF AT $\boldsymbol{\alpha} = (1, 0.2, 0.1)$.

Coalition $\mathcal{S}$	$v(\mathcal{S})$	$e(\mathcal{S}, \mathbf{x}^{\text{nu}})$	$e(\mathcal{S}, \mathbf{x}^{\text{shp}})$	$e(\mathcal{S}, \mathbf{x}^{\text{ave}})$	$e(\mathcal{S}, \mathbf{x}^{\text{ratio}})$
$\{1\}$	0.8	−0.075	−0.083	0.467	0.031
$\{2\}$	0	−0.075	−0.083	−0.333	−0.154
$\{3\}$	0	−0.05	−0.033	−0.333	−0.077
$\{1, 2\}$	0.9	−0.05	−0.067	0.233	−0.023
$\{1, 3\}$	0.8	−0.125	−0.117	0.133	−0.046
$\{2, 3\}$	0	−0.125	−0.117	−0.667	−0.231
$\{1, 2, 3\}$	1	0.0	0.0	0.0	0.0

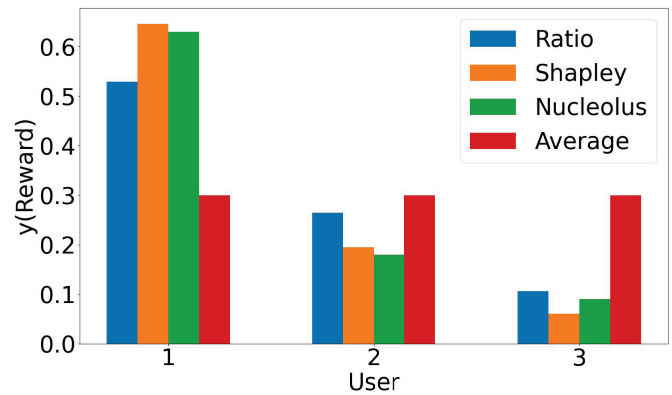

Fig. 3. Amount of reward for each player at $\boldsymbol{\alpha} = (1, 0.5, 0.2)$.

$$\boldsymbol{\theta}(\mathbf{x}^{\text{ratio}}) = (0.031, 0.0, -0.023, -0.046, -0.077, -0.154, -0.231)$$
$$\boldsymbol{\theta}(\mathbf{x}^{\text{shp}}) = (0.0, -0.033, -0.067, -0.083, -0.083, -0.117, -0.117)$$
$$\boldsymbol{\theta}(\mathbf{x}^{\text{nu}}) = (0.0, -0.05, -0.05, -0.075, -0.075, -0.125, -0.125)$$
$$\boldsymbol{\theta}(\mathbf{x}^{\text{ave}}) = (0.467, 0.233, 0.133, 0.0, -0.333, -0.333, -0.667)$$

Comparing the four excess vectors in the lexicographic order defined in Eq. (4), we obtain $\boldsymbol{\theta}(\mathbf{x}^{\text{ave}}) >_L \boldsymbol{\theta}(\mathbf{x}^{\text{ratio}}) >_L \boldsymbol{\theta}(\mathbf{x}^{\text{shp}}) >_L \boldsymbol{\theta}(\mathbf{x}^{\text{nu}})$.

As discussed in Sec. II, the Shapley value provides a fair imputation. However, this imputation may result in the user with a smaller area receiving too little compared to Nucleolus in terms of minimizing the maximum excess. In Example 1-1, $\mathbf{x}^{\text{ratio}}$ was an imputation and still belonged to the core, on the other hand, in Example 1-2, this imputation is not belonged.

This result indicates that a simple reward based on the area collected may not provide an imputation that satisfies everyone. Imputations belonging to the core of Shapley values and nucleolus may be effective in ensuring that everyone is not dissatisfied.

C. Reward distribution

These payoffs represent the proportion of each data provider's reward based on its area in this transaction. Next, we assume that two data consumers purchase the data ($r = 2$). Here, we set the fixed price of data $p = 0.15$, the revenue sharing coefficient $\eta = 0.3$, $\boldsymbol{\beta} = (0.8, 0.7)$, and the transmission cost $s = 0.1$. The total amount of rewards T passed by the data consumer to the data provider is 0.9 from Eq. (8).

Fig. 3 shows the amount of reward distribution $\boldsymbol{y}$ received by each data provider every payoff vector. The collected area of the data providers is the same as in Example 1-1. The distribution method determined by the ratio of $\boldsymbol{\alpha}$ is represented as Ratio, that determined by the Shapley value as Shapley, that determined by Nucleolus as Nucleolus, and that determined by the equal split as Average. In co-creative Digital Twin ecosystem, it is important to design incentives that allow for a return from data consumers to data providers. Fig. 3 shows that reward distribution from data consumers can be made in a way that satisfies data providers.

V. Conclusion

This paper deals with transactions of point cloud data in co-creative Digital Twin, modeled by coalitional game theory, and calculates the amount of reward allocated to each data provider. The area covered by each data provider is set as an element that characterizes a player, and the amount of reward is determined by the Shapley value and nucleolus of game theory. In the analysis of the numerical results, examples are presented focusing on the equal sharing method, the allocation based on the area covered, the Shapley value calculated based on the degree of contribution, and the allocation of the Nucleolus that minimizes the maximum excess of players.

The analysis showed that the equal sharing method and the allocation based on the area covered are not appropriate because they may lead to dissatisfaction among some data providers, depending on the extent of data collection. Since this paper only focuses on the area covered for point cloud data to determine the amount of reward, it is necessary to consider other factors such as the density of point clouds in the future. We also plan to consider the intent of data consumers when setting prices.

Acknowledgment

These research results were obtained from the commissioned research(No. JPJ012368C05601) by National Institute of Information and Communications Technology (NICT) , Japan. This work was supported by JSPS KAKENHI Grant Numbers 19K11947, 22K12015, 22H00247.

References

[1] L. Deren, Y. Wenbo, and S. Zhenfeng, "Smart city based on digital twins," in *Computational Urban Science*, vol. 1, no. 4, Mar. 2021.

[2] National Research Foundation, Singapore Government, "Virtual singapore," Online, https://www.nrf.gov.sg/programmes/virtual-singapore.

[3] "What is shizuoka prefecture's virtual shizuoka initiative?" Online, https://info.tokyo-digitaltwin.metro.tokyo.lg.jp/docs/kentoukai02 /dt_kentou_02_04.pdf.

[4] T. Ministry of Land, Infrastructure and Tourism, "Plateau," Online, https://www.mlit.go.jp/plateau/.

[5] T. Metropolitan, "Tokyo digital twin project," Online, https://info.tokyo-digitaltwin.metro.tokyo.lg.jp/.

[6] K. Kanai, T. Yamazaki, S. Miyata, H. Kanemitsu, A. Mine, S. Mori, and H. Nakazato, "Research and development of co-creating digital twins using web3 technologies to accelerate beyond 5g," *IEICE Technical Report*, vol. 122, no. 269, pp. 1–6, 11 2022, CS2022-48.

[7] Waseda University, "Towards the realisation of a co-creative digital twin," Online, https://www.waseda.jp/top/news/83150.

[8] J. Watanabe, S. Miyata, and K. Kanai, "Data transaction model that takes into account the characteristics of point cloud data for the realisation of a co-created digital twin," *IEICE Technical Report*, 9 2023.

[9] A. Okada, *Game Theory: 3rd Edition*. Tokyo: Yuhikaku Publishing Co., Ltd., 2021.

[10] V. S. Dasari, B. Kantarci, M. Pouryazdan, L. Foschini, and M. Girolami, "Game theory in mobile crowdsensing: A comprehensive survey," *Sensors*, vol. 20, no. 7, p. 2055, 2020.

[11] C. Jiang, L. Gao, L. Duan, and J. Huang, "Economics of peer-to-peer mobile crowdsensing," *2015 IEEE Global Communications Conference (GLOBECOM)*, pp. 1–6, 2015.

[12] C. Jiang, L. Gao, L. Duan, and J. Huang, "Scalable mobile crowdsensing via peer-to-peer data sharing," *IEEE Transactions on Mobile Computing*, vol. 17, no. 4, pp. 898–912, 2018.

[13] C. Jiang, L. Gao, L. Duan, and J. Huang, "Data-centric mobile crowdsensing," *IEEE Transactions on Mobile Computing*, vol. 17, no. 6, pp. 1275–1288, 2018.

[14] H. Hotoyama, J. Watanabe, S. Miyata, K. Kanai, and T. Yamazaki, "Data trading model based on coalitional game and its numerical analysis towards co-creation digital twins," *IEICE Technical Report*, vol. 122, no. 407, pp. 299–304, 3 2023.

[15] M. Nakayama, *Fundamentals and applications of co-operative games*. Tokyo: Keiso Shobo, 2012.

[16] L. S. Shapley, "A value for n-person games," pp. 307–317, 1953.

[17] S. C. Littlechild and G. Owen, "A simple expression for the shapley value in a special case," *Management Science*, vol. 20, pp. 370–372, 1973.

[18] J. Ni, X. Lin, Q. Xia, and X. S. Shen, "Dual-anonymous reward distribution for mobile crowdsensing," *2017 IEEE International Conference on Communications (ICC)*, pp. 1–6, 2017.

[19] T. Oishi and M. Nakayama, "Anti-dual of economic coalitional tu games," *JAPANESE ECONOMIC REVIEW*, vol. 60, no. 4, pp. 560–566, 12 2009.

[20] J. Liu, L. Gao, T. Wang, X. Zeng, W. Lu, and Y. Zhong, "Crowsourcing: A novel approach to organizing wifi community networks," *2018 16th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, pp. 1–8, 2018.